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**Abstract**

This paper describes the elements of a social network. It is the result of a study of the Italian social network.

A social network is a set of people or groups of people with some pattern of contacts or interactions between them. The elements of a social network are the nodes and the ties. Nodes are the individual actors within the networks, and ties are the relationships between the actors. There can be many kinds of ties between the nodes. Research in a number of academic fields has shown that social networks operate on many levels, from families up to the level of nations, and play a critical role in determining the way problems are solved, organizations are run, and the degree to which individuals succeed in achieving their goals.

The social network is a theoretical construct useful in the social sciences to study relationships between individuals, groups, organizations, or even entire societies (social units, see differentiation). The term is used to describe a social structure determined by such interactions. The ties through which any given social unit connects represent the convergence of the various social contacts that it has. This theoretical approach is necessary to analyze social institutions in network terms. Consequently, it is different from traditional sociological study, which assumes that it is possible to understand social units deductively without empirical investigation of the relations between them.

The study of social networks is also known as social network analysis (SNA) or structural analysis. The visual representation of social networks is important to understand the network data and to convey the results of the analysis. Many of the concepts and terminology of social networks are intuitive. The network researcher Linton Freeman wrote that "Social network analysis is primarily an intuitive approach that lends itself more easily to visual presentation than to mathematical manipulation."

## Graph theory and social networks

Graph theory is the mathematical study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices (also called nodes or points) which are connected by edges (also called links or lines). A distinction is made between undirected graphs, where edges link two vertices symmetrically, and directed graphs, where edges link two vertices asymmetrically. Graphs are one of the principal objects of study in discrete mathematics.

The terms used in graph theory are standardized. A graph is an ordered pair $G = (V, E)$ comprising a set $V$ of vertices or nodes together with a set $E$ of edges or lines, which are 2-element subsets of $V$ (i.e., an edge is related with two vertices, and the relation is represented as an unordered pair of the vertices with respect to the particular edge). To avoid ambiguity, this type of object may be called precisely an undirected simple graph.

In edge and vertex counting, the number of edges may be zero, but to be nontrivial the graph must have at least one vertex. Moreover, two edges are said to be adjacent if they are connected to the same vertex, and two vertices are said to be adjacent if they are connected to the same edge. The edge connecting vertices $u$ and $v$ is written $uv$. To avoid ambiguity, this type of object may be called precisely an undirected simple graph.

In mathematics, graphs are useful in geometry and certain parts of topology such as knot theory. Algebraic graph theory has close links with group theory. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another; see graph (mathematics) for more detailed definitions and for other variations in the types of graph that are commonly considered. Graphs are one of the prime objects of study in discrete mathematics.

### Definitions

Definitions in graph theory vary. The following are some of the more basic ways of defining graphs and related mathematical structures.

In one restricted but very common sense of the term, a graph is an ordered pair $G = (V, E)$ comprising:

- $V$, a set of vertices (also called nodes or points);
- $E \subseteq \{\{x, y\} \mid x, y \in V\}$, a set of edges (also called links or lines), which are unordered pairs of vertices (that is, an edge is associated with two distinct vertices).